Electric dipole moments of leptons in the presence of Majorana neutrinos

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We calculate the two-loop diagrams that give a nonzero contribution to the electric dipole moment d_l of a charged lepton l due to possible Majorana masses of neutrinos. Using the example with one generation of the standard model leptons and two heavy right-handed neutrinos, we demonstrate that the nonvanishing result for d_l first appears in order $O(m_l m_\nu^2 G_F^2)$, where m_ν is the mass of the light neutrino and the see-saw type relation is imposed. This effect is beyond the reach of presently planned experiments.

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I. INTRODUCTION

Recent discovery of CP violation in the neutral B-meson system [1,2] is in perfect accord with the CP violation observed in K mesons [3]. Both results are explained within the minimal model of CP violation, known as the Kobayashi-Maskawa model [4]. It links CP-violation in K and B mesons to a single CP-violating invariant of the Kobayashi-Maskawa matrix (KM) in the quark sector, $\mathrm{Im}(V_{tb}V_{td}^*V_{cd}V_{cb}^*) \simeq 3 \times 10^{-5}$. This combination, as well as θ_{QCD}, are the only sources of CP violation in the Standard Model.

An independent piece of experimental information about CP violation comes from the searches of the electric dipole moments (EDMs) of neutrons [5] and heavy atoms [6,7]. It is well known that the Kobayashi-Maskawa model predicts extremely small EDMs. Indeed, the necessity of four electroweak vertices requires from any diagram capable of inducing an EDM of a quark to have at least two loops. Moreover, it turns out that all EDMs or color EDMs of quarks vanish exactly at the two-loop level [8], and only three-loop diagrams survive [9,10], producing a tiny number of order $10^{-34}e$cm.

The only relevant operator that is not zero at the two-loop order is the Weinberg operator [11], but its numerical value turns out to be also extremely small. Possible enhancement comes from the large distance effects, that could lead to a KM-generated EDM of the neutron of order $10^{-32}e$ cm [12,13], which is still six to seven orders of magnitude smaller than the current experimental limit. The KM phase in the quark sector induces the EDM of a lepton via a diagram with a closed quark loop, but a nonvanishing result appears first at a four-loop level [14] and therefore is even more suppressed.

The suppressed values of EDMs from the KM model together with enormous accuracy of EDM experiments produce stringent constraints on possible flavour-diagonal sources of CP violation, notably on CP-odd combinations of the soft-breaking parameters in the supersymmetric extensions of the standard model (SM).

On the other hand, the KM phase cannot be the only source of CP violation in nature. Dynamical generation of the baryon asymmetry of the Universe (BAU) requires presence of an additional source(s) of CP violation in nature. One of the most appealing scenarios for BAU is leptogenesis [15] where a nonzero lepton number is generated from the out-of-equilibrium decay of heavy right-handed Majorana neutrinos with CP violation coming from the Yukawa sector of the theory. Subsequent sphaleron transitions [16] transform half of the initial lepton asymmetry into BAU.

The attractiveness of leptogenesis is in its simplicity, relative freedom from the low-energy constraints, and in recent experimental results that indicate large mixing angles in the lepton sector [17].

The complex phases in the lepton mixing suggested by leptogenesis at some level will induce d_e, the electron EDM, as well as EDMs of other charged leptons. Various calculations of d_e and d_μ were performed in the supersymmetric case under certain assumption about the soft-breaking parameters [18], while a nonsupersymmetric case remains poorly explored.

We note that manifestations of Majorana phases in CP-violating phenomena have been explored in [19]. The EDM is an additional observable arising due to those phases. Compared with the KM model, where three generations and at least four loops are necessary to generate a lepton EDM, the addition of Majorana phases enhances the effect. However, as we will see, it remains very small.

In this paper, we present a systematic analysis of the two-loop diagrams that lead to the EDMs of charged leptons through interactions with two additional heavy neutrinos, in a one generation model suggested by a minimal leptogenesis scenario. These diagrams can be divided into two major classes. The first class where the lepton number is conserved along the fermion line vanishes identically, as the same arguments that lead to $d_q = 0$ at two loops [8] apply (see Fig. 1).

The second class of diagrams (Fig. 2) has no analogues in the quark sector: due to the existence of the Majorana component of the neutrino mass matrix, a combination of $\Delta L = 1$ and $\Delta L = -1$ transitions is possible. The existence of this additional class of diagrams for leptons was pointed out in Ref. [20]. It leads to a nonzero electron EDM (d_e) even at the two-loop level, but up to now there has not been any serious attempt to calculate the size of d_e induced by this class of diagrams. (An estimate of d_e due to CP violation in the sector of heavy Majorana neutrino was presented in Ref. [21]. However, this result is not satisfactory, as it suggests that d_e does not vanish in the limit of infinitely heavy right-handed neutrinos.)

The purpose of this work is to give a detailed calculation of d_e due to the CP violation in the lepton sector in the presence of Majorana masses for neutrinos and obtain a prediction for the electron EDM in the see-saw model of the neutrino mass sector. A similar mechanism can of course also lead to the EDM of the muon.

We find that two-loop diagrams give nonvanishing results for the d_e. However, EDMs are very much suppressed by the smallness of the neutrino mass as they first appear in $O(m_e m_\nu^2 G_F^2)$ order, if the see-saw type relation between m_ν, Dirac mass and heavy Majorana mass is imposed. Thus, for any phenomenologically motivated choice of m_ν the result for d_e (and d_μ) turns out to be much smaller than current or projected experimental sensitivity to EDMs. We also note that the fine-tuning in the neutrino mass sector may lead to up to 10 orders of magnitude enhancement in d_e, which is still not enough to bring it within the experimental reach in the near future.

II. DESCRIPTION OF THE MODEL

We take one standard model generation: $\binom{\nu_L}{e_L}$, e_R, and two singlet heavy neutrinos $N_{1,2}$. The latter do not participate in electroweak interactions; in particular, the charged current sector is described by the Lagrangian

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} [\bar{\nu}_L \not{W}^+ e_L + (\text{H.c.})] \qquad (1)$$

The mass sector Lagrangian for fermions is

$$\begin{aligned} -\mathcal{L}_M = {} & m_e(\bar{e}_L e_R + \bar{e}_R e_L) + \frac{M_1}{2}(\bar{N}_1^c N_1 + \bar{N}_1 N_1^c) \\ & + \frac{M_2}{2}(\bar{N}_2^c N_2 + \bar{N}_2 N_2^c) + m_1(e^{i\phi_1}\bar{N}_1\nu_L \\ & + e^{-i\phi_1}\bar{\nu}_L N_1) + m_2(e^{i\phi_2}\bar{N}_2\nu_L + e^{-i\phi_2}\bar{\nu}_L N_2). \end{aligned} \qquad (2)$$

Here $\psi^c \equiv \gamma_0 C\psi^*$; $M_{1,2}$ and $m_{1,2}$ are defined in terms of real positive Yukawa couplings $y_{1,2}$ and the electroweak vacuum expectation value v,

$$m_{1,2} \equiv \frac{y_{1,2} v}{\sqrt{2}}. \qquad (3)$$

We use the freedom of phase choice for ν_L and $e_{R,L}$ to redefine

$$\nu_L \rightarrow e^{-i\phi_2}\nu_L. \qquad (4)$$

We see that there is only one physical CP violating phase $\eta \equiv \phi_1 - \phi_2$.

Before we explore the physical manifestation of η, we determine the mass eigenstates of neutrinos. We will use them to compute the EDM of the electron. We use the identity

$$\bar{\nu}_L N = \frac{1}{2}(\bar{\nu}_L N + \bar{N}^c \nu_L^c) \qquad \bar{N}\nu_L = \frac{1}{2}(\bar{N}\nu_L + \bar{\nu}_L^c N^c) \qquad (5)$$

to rewrite the neutrino mass matrix as

$$-\mathcal{L}_{M,\nu} = \frac{1}{2}(\bar{\nu}_L, \bar{N}_1^c, \bar{N}_2^c)\mathcal{M}\begin{pmatrix} \nu_L^c \\ N_1 \\ N_2 \end{pmatrix} + (\text{H.c.})$$

$$\mathcal{M} \equiv \begin{pmatrix} 0 & m_1 e^{i\eta} & m_2 \\ m_1 e^{i\eta} & M_1 & 0 \\ m_2 & 0 & M_2 \end{pmatrix} \qquad (6)$$

We use $M_{1,2} \gg m_{1,2}$ to approximately diagonalize the neutrino mass matrix. We define neutrino mass eigenstates: light ν and heavy $n_{1,2}$,

$$\begin{pmatrix} \nu_L^c \\ N_1 \\ N_2 \end{pmatrix} = V \begin{pmatrix} \nu^c \\ n_1 \\ n_2 \end{pmatrix} \qquad (7)$$

where V is the matrix diagonalizing $\mathcal{M}$. Approximately,

$$V = \begin{pmatrix} 1 & \frac{m_1 e^{i\eta}}{M_1} & \frac{m_2}{M_2} \\ -\frac{m_1 e^{i\eta}}{M_1} & 1 & 0 \\ -\frac{m_2}{M_2} & 0 & 1 \end{pmatrix}$$

$$V^T \mathcal{M} V = \begin{pmatrix} -\frac{m_1^2 e^{2i\eta}}{M_1} - \frac{m_2^2}{M_2} & o_2 & o_2 \\ o_2 & M_1 + o_1 & o_1 \\ o_2 & o_1 & M_2 + o_1 \end{pmatrix} \qquad (8)$$

where we denote small corrections by $o_n \equiv \mathcal{O}[m(\frac{m}{M})^n]$. Two things are worth noting:

- Heavy neutrinos $n_{1,2}$ participate in charged current interactions described by Eq. (1), through their presence in ν_L, Eq. (7):

$$\nu_L^c \simeq \nu^c + \frac{m_1 e^{i\eta}}{M_1} n_1 + \frac{m_2}{M_2} n_2. \qquad (9)$$

- The CP violating phase enters through the complex mass of ν and through the charged current interaction of n_1.

The Majorana mass of the light (active) neutrino is given by the following relation,

$$m_\nu = \left| \frac{m_1^2 e^{2i\eta}}{M_1} + \frac{m_2^2}{M_2} \right|. \qquad (10)$$

Since experiments with light neutrinos give information about m_ν, it is convenient to keep this parameter fixed, while allowing $m_{1,2}$, $M_{1,2}$ and η to vary. In doing this, we would have to distinguish two possibilities.

- See-saw relation, when both contributions in (10) are on the order or smaller than m_ν,
- Cancellation of two terms in m_ν which we would term as a "fine-tuned case", although it could be a result of some symmetry that suppresses the determinant of the mass matrix (6). For this cancellation to happen, one has to have $m_1^2/M_1 \simeq m_2^2/M_2$ and phase η close to $\pi/2$.

We would like to note that theoretically one could have a fine-tuned case even for $m_{1,2} \sim M_{1,2}$. This, however, would also result in large admixtures of heavy n_1 and n_2 neutrinos in the original left-handed neutrino. In a more realistic model with three active neutrino species, such admixtures could lead to a nonuniversality of charged currents, modification of the invisible Z decay width, etc., With the accuracy of electroweak data, we impose the bound on the size of the mixing angle:

$$\frac{m_1}{M_1}, \frac{m_2}{M_2} \lesssim \mathcal{O}\left(\frac{1}{10}\right). \qquad (11)$$

III. COMPUTATION OF THE ELECTRON EDM AT TWO LOOPS

To define the electron EDM, consider the general matrix element of an electromagnetic current between spin 1/2 fermions,

$$\bar{u}_f(p)\Big\{F_1(t)\gamma_\mu - \frac{i}{2m}F_2(t)\sigma_{\mu\nu}q^\nu + \frac{1}{m}F_3(t)q_\mu + \gamma_5\Big[G_1(t)\gamma_\mu - \frac{i}{2m}G_2(t)\sigma_{\mu\nu}q^\nu + \frac{1}{m}G_3(t)q_\mu\Big]\Big\} \times u_i(p+q), \qquad (12)$$

with $t \equiv q^2$. The EDM d_e of an electron of charge e is given by

$$d_e = -\frac{ie}{2m_e}G_2(0). \qquad (13)$$

The most notable property of all diagrams in Figs. 1 and 2, potentially contributing to the CP-odd amplitude, is the effective antisymmetrization over the neutrino propagation, f_1 and f_2. This is because the CP-odd part of a diagram with the selection of eigenstates f_1 and f_2 is always *opposite* to the CP-odd part of the diagram with interchanged flavours. As a result of this antisymmetrization, in the expansion over small q the first class of

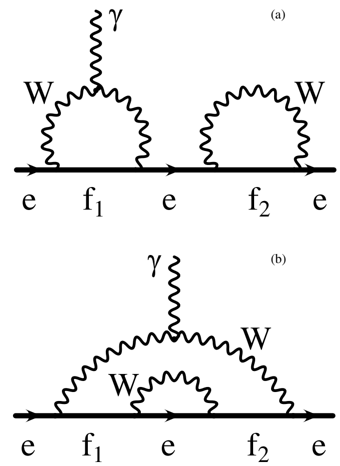

FIG. 1. Examples of two-loop diagrams that give zero electric dipole moment of the electron. The direction of the arrows remains the same on all fermion propagators and the external photon can be attached to any charged line.

diagrams, Fig. 1(a) and 1(b), turns out to be proportional to the cube of the photon momentum $O(q^3)$, while $G_2(0)$ vanishes identically. A more detailed explanation of the cancellation of EDMs can be found in Refs. [8,11,14].

Within the model we are considering, the nonvanishing diagrams contributing to d_e are shown in Fig. 2. Their evaluation is simplified if we assume the most natural mass hierarchy, $\frac{m_{1,2}^2}{M_{1,2}} \ll m_e \ll M_W \ll M_{1,2}$. We use the notation $M = \frac{M_1+M_2}{2}$ and $\Delta M = M_2 - M_1$ and assume $|\Delta M| \ll M$ for calculational convenience.

First, we treat the case when $f_{1,2}$ are both heavy (some details of derivations of these results are given in the Appendix),

$$\Delta d_e(\text{heavy-heavy}) = e\left(\frac{G_F}{16\pi^2}\right)^2 m_e \frac{\Delta M}{M}\frac{m_1^2}{M}\frac{m_2^2}{M} \times M^{-4\epsilon}\left(\frac{16}{3\epsilon} - \frac{364}{9} + \frac{112}{27}\pi^2\right)\sin 2\eta. \qquad (14)$$

Next, we obtain the contribution when one of f_i is the light state ν,

$$\Delta d_e(\text{heavy-light}) = e\left(\frac{G_F}{16\pi^2}\right)^2 m_e \frac{\Delta M}{M}\frac{m_1^2}{M} \times \frac{m_2^2}{M}M^{-6\epsilon}M_W^{2\epsilon}\left(-\frac{16}{3\epsilon} + \frac{104}{9}\right)\sin 2\eta. \qquad (15)$$

The sum of both contributions is finite,

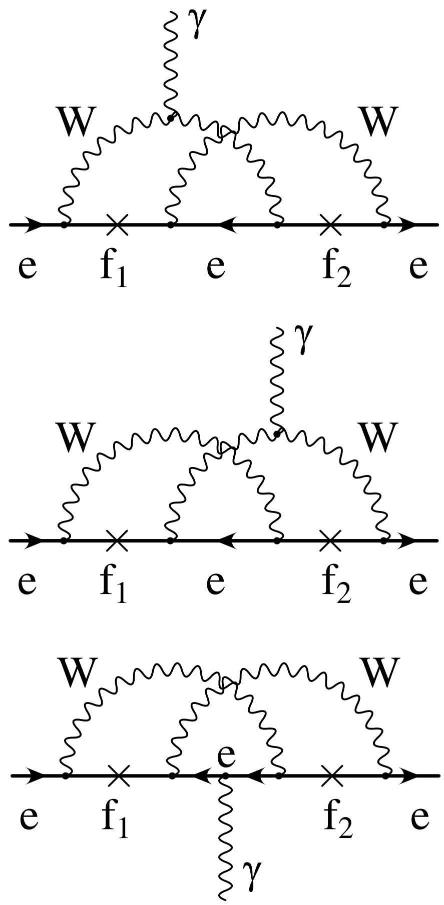

FIG. 2. Contributions to the electron EDM in a model with Majorana masses of neutrinos. $f_{1,2}$ denote all possible neutrinos (see text). Crosses denote insertions of lepton number violating mass parameters. Note that the direction of the internal electron line is opposite to the external ones.

$$\begin{aligned} d_e &= \Delta d_e(\text{heavy-heavy}) + \Delta d_e(\text{heavy-light}) \\ &= e\left(\frac{G_F}{16\pi^2}\right)^2 m_e \frac{\Delta M}{M}\frac{m_1^2}{M}\frac{m_2^2}{M}\left(\frac{32}{3}\ln\frac{M}{M_W} - \frac{260}{9}\right. \\ &\quad \left. + \frac{112}{27}\pi^2\right)\sin 2\eta. \end{aligned} \tag{16}$$

Our answer shows that the electron EDM first appears at $O(m_e m_1^2 m_2^2 M^{-2} G_F^2)$ level, which is essentially the same as $O(m_e m_\nu^2 G_F^2)$ if the see-saw relations $m_{1,2}^2/M \sim O(m_\nu)$ are in place.

In a slightly different model, with two generations of active neutrinos with masses $m_{\nu 1}$ and $m_{\nu 2}$ there is an additional possibility of f_1 and f_2 both being light. It is easy to see, however, that in the case of (light-light) diagram the contribution to d_e is parametrically much smaller than Eq. (16), being suppressed by four powers of light neutrino mass $m_{\nu 1} m_{\nu 2}(m_{\nu 1}^2 - m_{\nu 2}^2)$. Thus, the generalization to the case of three generations would not bring any parametric change to the size of d_e. Fixing m_ν, we can also allow $m_{1,2}$ and $M_{1,2}$ to vary. If $M_{1,2}$ becomes smaller than the electroweak scale, d_e acquires additional suppression by M^2/M_W^2. This proves that (16) with the chosen hierarchy of scales represents the largest possible contribution to d_e at fixed m_ν.

IV. DISCUSSION AND CONCLUSIONS

What is the largest numerical value of the answer (16)? For the see-saw type relation, in the absence of substantial cancellations between the two terms in (10), we can use the following inequality

$$\left|\frac{m_1^2}{M}\frac{m_2^2}{M}\sin 2\eta\right| \lesssim m_\nu^2. \tag{17}$$

This leads to an extremely tight theoretical bound on the possible size of the EDM,

$$|d_e| \lesssim e\left(\frac{G_F}{16\pi^2}\right)^2 m_e m_\nu^2 \frac{|\Delta M|}{M}\left(10.7 \times \ln\frac{M}{M_W} + 12.1\right), \quad \text{see-saw case.} \tag{18}$$

Using $|\Delta M| \sim M \sim 10^{16}$ GeV, and the Particle Data Group book [22] bound on the mass of the electron neutrino, $m_\nu < 3$ eV, we arrive at our final numerical result for the see-saw case,

$$|d_e| < 1.5 \times 10^{-43} e\ \text{cm, see-saw case.} \tag{19}$$

Not only is this number much smaller than the most optimistic accuracy of future electron EDM searches [23], but also it is significantly smaller than EDMs induced by the KM phase from the quark sector.

The fine-tuned case deserves special consideration, as the numerical answer for d_e can be significantly larger than (19). Introducing another angle $\delta = \pi/2 - \eta$, which has to be small for the fine-tuning of m_ν to happen, we rewrite the light neutrino mass squared as

$$m_\nu^2 = \frac{1}{M^2}[(m_1^2 - m_2^2)^2 + 4m_1^2 m_2^2 \delta^2]. \tag{20}$$

On the other hand, the answer for EDM contains a factor $\frac{m_1^2 m_2^2}{M^2}\sin(2\eta) = 2\frac{m_1^2 m_2^2}{M^2}\delta$, which in view of Eq. (20) can be limited as

$$\left|\frac{m_1^2 m_2^2}{M^2} 2\delta\right| < \frac{m_1 m_2}{M} m_\nu \lesssim (0.1 \times 500\ \text{GeV}) m_\nu. \tag{21}$$

In this expression, we have used (11) and the upper bound on $m_{1,2}$ taken to be ~500 GeV. The latter follows from the condition that Yukawa sector remains perturbative, i.e. $y_{1,2} \lesssim O(1 - 10)$. Comparing (17) and (21), we observe that the fine-tuned case may lead to up to 10^{10} enhancement relative to the see-saw case and the electron EDM may reach a value of

$$d_e \lesssim 10^{-33} e \text{ cm}, \text{ fine-tuned case.} \tag{22}$$

This value is still far from the existing or projected experimental accuracy but is much larger than the electron EDM induced by the KM phase.

In some models the effects of the CP-odd electron-nucleon interaction, $C_S \bar{N} N \bar{e} i\gamma_5 e$ may dominate over the electron EDM contribution [24] in the atomic (molecular) EDM. An analysis of possible two-loop diagrams for C_S shows that in the model with CP violation coming from Majorana neutrinos this is not the case, and the contribution of $C_S(\eta)$ to atomic EDM is smaller than (19) for the see-saw case and (22) for the fine-tuned case.

To summarize, we have calculated the contributions of the two-loop diagrams to the EDM of the electron in nonsupersymmetric models with CP violation in the lepton sector and with Majorana masses for neutrinos. We notice that the nonzero result for d_e can be obtained in a rather minimalistic way: with one generation of SM leptons and two right-handed neutrinos. In terms of the right-handed mass M and Dirac mass m_D, the nonzero result appears in order $O(m_e m_D^4 M^{-2} G_F^2)$, which is the largest possible result given the symmetries of the model. If the smallness of the light neutrino mass is achieved via usual see-saw relation without any fine-tuning, this answer is equivalent to $O(m_e m_\nu^2 G_F^2)$ and numerically is extremely small. Numerical smallness of this result stems from the smallness of m_ν, and the parametric suppression of EDM is very similar to the suppression of *e.g.* the SM amplitude for $\mu \to e\gamma$ decay. In the fine-tuned case, when the smallness of the light neutrino mass is achieved via near perfect cancellation of two contributions, the result for d_e may become larger by many orders of magnitude, but still smaller than about $10^{-33} e$ cm, and therefore much smaller than the sensitivity of any EDM experiment in the foreseeable future.

We conclude that the KM-type models with Majorana neutrino masses do not have any impact on the EDM searches. Therefore, the only options of searching for CP-violation in the neutrino sector suggested by the leptogenesis scenario are the CP asymmetries in neutrino oscillations and not EDMs. Conversely, possible positive results from the future electron EDM searches could be an indication of leptogenesis combined with the soft supersymmetry breaking.

ACKNOWLEDGMENTS

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APPENDIX A: DETAILS OF THE CALCULATION OF TWO-LOOP DIAGRAMS

Here we give intermediate results for the sum of the three diagrams in Fig. 2, before we add and simplify contributions from $n_{1,2}$.

For the heavy-heavy case we get

$$\begin{aligned}\Delta d_e(\text{heavy-heavy}) &= -\frac{ie^5}{2m_e s_W^4}\frac{1}{(16\pi^2)^2}\left(\frac{m_1 m_2}{M_1 M_2}\right)^2 (e^{2i\eta} \\ &\quad - \text{complex conjugate}) \cdot (M_2 - M_1) \\ &\quad \times \frac{M m_e^2}{M_W^4} M^{-4\epsilon}\left(\frac{1}{6\epsilon} - \frac{91}{72} + \frac{7}{54}\pi^2\right) \\ &\simeq \left(\frac{G_F}{16\pi^2}\right)^2 m_e e \frac{\Delta M m_1^2 m_2^2}{M^3} M^{-4\epsilon}\left(\frac{16}{3\epsilon} \right. \\ &\quad \left. - \frac{364}{9} + \frac{112}{27}\pi^2\right)\sin 2\eta. \end{aligned} \tag{A1}$$

For the heavy-light case we find

$$\begin{aligned}\Delta d_e(\text{heavy-light}) &= -\frac{ie^5}{2m_e s_W^4}\frac{1}{(16\pi^2)^2}\frac{m_e^2}{M_W^4} \\ &\quad \cdot \left[\frac{m_1^2}{M_1^2} e^{2i\eta} f\left(-\frac{m_2^2}{M_2}, M_1\right) \right. \\ &\quad + \frac{m_2^2}{M_2^2} f\left(-\frac{m_1^2}{M_1} e^{-2i\eta}, M_2\right) \\ &\quad \left. - \text{complex conjugate}\right], \end{aligned} \tag{A2}$$

where the function f arises from the evaluation of the diagrams without coupling constants,

$$\begin{aligned} f(m_\nu, M_i) &\equiv m_\nu M_i M_W^{-4\epsilon}\left(-\frac{1}{12\epsilon}\ln\frac{M_i^2}{M_W^2} - \frac{5}{16\epsilon} + \frac{13}{72}\right. \\ &\quad \left. \times \ln\frac{M_i^2}{M_W^2} + \frac{1}{8}\ln^2\frac{M_i^2}{M_W^2} - \frac{31}{48} + \frac{\pi^2}{36}\right). \end{aligned} \tag{A3}$$

Here m_ν and M_i denote a light and a heavy neutrino mass insertion. Working to first order in the heavy neutrino mass splitting ΔM, we get

$$\begin{aligned}\Delta d_e(\text{heavy-light}) &= -\frac{ie^5}{2m_e s_W^4}\frac{1}{(16\pi^2)^2}\frac{m_e^2}{M_W^4}\cdot\frac{m_1^2 m_2^2}{M^3} i\frac{\Delta M}{16} M_W^{-4\epsilon}\left(-\frac{16}{3\epsilon} + \frac{104}{9} + 16\ln\frac{M^2}{M_W^2}\right)\sin 2\eta \\ &= \left(\frac{G_F}{16\pi^2}\right)^2 m_e e \frac{\Delta M m_1^2 m_2^2}{M^3} M_W^{-4\epsilon}\left(-\frac{16}{3\epsilon} + \frac{104}{9} + 16\ln\frac{M^2}{M_W^2}\right)\sin 2\eta \end{aligned} \tag{A4}$$

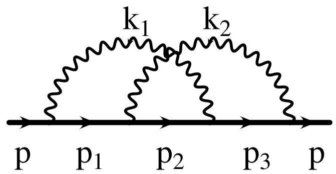

FIG. 3. Momentum assignment for the EDM calculation.

We see that in the sum of contributions in (A1) and (A4), the divergences cancel and we obtain the final result, Eq. (16).

APPENDIX B: ASYMPTOTIC EXPANSIONS

In this appendix, we show how the method of asymptotic expansions [25] is used to calculate the contributing diagrams for the electric dipole moment.

Two-loop Feynman diagrams containing more than one mass scale are difficult to solve analytically. Analytic expansions in small parameters, such as the ratio of lepton and weak boson masses, are used to reduce the calculations to integrals involving only one mass scale. The results are integrals which have analytic solutions. As an example of this technique, we use Feynman diagrams from Fig. 2 containing one light neutrino, f_1, of mass m_ν and one heavy neutrino, f_2, of mass M.

The momentum assignments for the calculation are illustrated in Fig. 3. Conservation of momentum at each vertex requires $p_1 = p + k_1$, $p_2 = p + k_1 + k_2$ and $p_3 = p + k_2$.

In order to perform the expansions, all mass scales must be identified. For our calculation, the mass scales involved are $M \gg M_W \gg m_e \gg m_\nu$.

Once the mass scales are determined, the integral volumes are divided into regions so that the momentum flow through the internal lines are on the order of one of the mass scales. With one light and one heavy neutrino, there are four scenarios to be considered:

(1) $k_1, k_2 \ll M, M_W$
(2) $k_2 \sim M, k_1 \ll M, M_W$
(3) $k_1, k_2 \sim M$
(4) $k_1, k_2 \sim M, k_1 + k_2 \sim m_e$

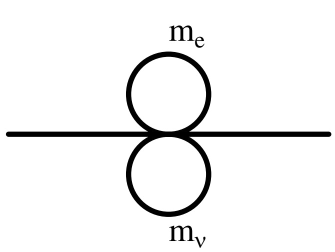

FIG. 4. Reduction of a two-loop diagram to two one-loop diagrams for $k_1, k_2 \ll M, M_W$. The solid lines represent massive propagators.

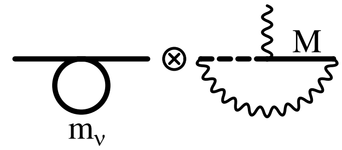

FIG. 5. Reduction of a two-loop diagram to two one-loop diagrams for $k_2 \sim M$, $k_1 \ll M, M_W$. The dashed line represents a massless propagator.

Within each region, the appropriate propagators can be expanded in a Taylor series to reduce the number of mass scales in the integrals.

1. $k_1, k_2 \ll M, M_W$

In this momentum region, the heavy neutrino and the W boson propagators can be expanded in a Taylor series. For example, the heavy neutrino propagator can be written as

$$\frac{1}{k^2 + M^2} = \frac{1}{M^2} \sum_{n=0}^{\infty} \left(\frac{k^2}{M^2}\right)^n \tag{B1}$$

since $k \ll M$.

The Taylor expansion reduces the diagram in Fig. 3 to that of Fig. 4. The two-loop diagram is reduced to the product of two one-loop diagrams. Each one-loop diagram has one mass scale and has an analytic solution.

2. $k_2 \sim M, k_1 \ll M, M_W$

When one momentum is large and the other is small, the propagators are expanded in all momenta except for the large momentum. This reduces the calculation to the diagrams in Fig. 5, a product of two one-loop diagrams. Clearly, the first diagram has only one mass scale involved. In the second diagram, partial fractions are used to further separate the mass scales. In particular, we use the identity

$$\frac{1}{k^2 + M^2} \frac{1}{k^2 + M_W^2} = \frac{1}{M_W^2 - M^2}\left(\frac{1}{k^2 + M^2} - \frac{1}{k^2 + M_W^2}\right) \tag{B2}$$

The resulting diagrams can be computed analytically.

3. $k_1, k_2 \sim M$

If both momenta are large, a reduction is made to the form of Fig. 6. Notice however, that there are still two

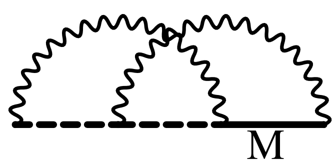

FIG. 6. Taylor Expansion for $k_1, k_2 \sim M$.

mass scales in the diagram. Thus, we use the assumption that $M \gg M_W$ for further expansion to reduce the diagram down to a one mass scale diagram.

4. $k_1, k_2 \sim M, k_1 + k_2 \sim m_e$

The last case reduces to Fig. 7 but does not contribute to the d_e in the leading order.

After integration over the initial ranges is performed, the contributions from all momentum regions are summed to produce Eq. (15).

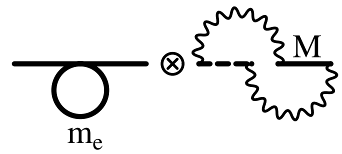

FIG. 7. Taylor expansion for the case $k_1 + k_2 \ll M$.

Using a similar analysis, Eq. (14) was computed for the diagrams with two heavy neutrinos.

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